Eton College King's Scholarship Examination 2023

MATHEMATICS A

(**One and a half hours**)

Candidate Number:……………………………………………………

__Please write your candidate number on EVERY sheet__.

Please answer on the paper in the spaces provided.

This paper is divided into two sections:
> *Section I (Short-answer questions) – 50 marks available*
> *Section II (Extended questions) – 50 marks available*

Answer all of Section I and as many questions as you can from Section II.

The marks for each part of each question are given in square brackets.

Show all your working.

No diagram is drawn to scale.

Neither calculators nor protractors may be used.

__ADDITIONAL MATERIALS__: NONE

Do not turn over until told to do so.

Section I: Short-answer questions (50 marks)

1) A fair, six-sided dice is thrown. Accurately illustrate, by using arrows and stating the letter, where the following events would be placed on the probability scale below.

   A: A number less than 8 is thrown.

   B: A '3' is thrown.

   C: An even number is thrown.

   0 _____ 1

   [3]

2) A class consists of 10 students. Here are the test marks split by gender.

Girls:	5	3	10	2	7	3
Boys:	2	5	9	3		

   a) Write down the modal mark of the class.

   [1]

   b) Work out the difference between the median mark of the girls and the median mark of the boys.

   [2]

   c) Work out the class mean mark.

   [2]

3) Find the value of the following, giving your answers as **reduced, mixed fractions.**

a) $3\frac{1}{6} + 1\frac{1}{4} \times 3\frac{3}{5}$

[3]

b) $56\frac{11}{18} - 72\frac{7}{12}$

[3]

4) Find the value of the following, giving your answers as **decimals.**
    a) 0.037×0.0078

                                                                        [3]

    b) $0.001311 \div 0.023$

                                                                        [3]
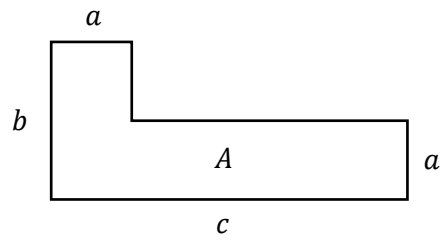
5)

a) Fully simplify the following expressions.

i) $\frac{9}{4}x - \frac{3x}{8}$

[1]

ii) $\left(\frac{a^2}{b} \div \frac{b}{c}\right) \times \left(b^2 \div \frac{a}{c}\right)$

[2]

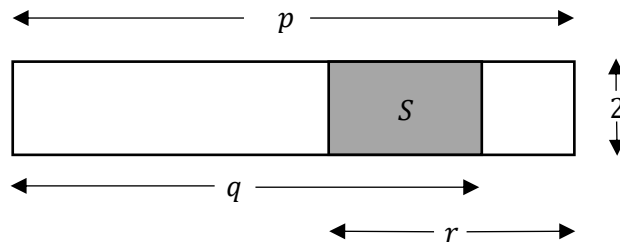b) What do you need to subtract from $7xy$ to get $8yx$?

[1]

6)

a)

i) Find a **simplified** formula for the area, A, of this shape in terms of a, b and c.

[2]

ii) Find the value of A when $a = 0.5$, $b = 3$ and $c = 7$, giving your answer as **a decimal.**

[2]

b)

i) Find a formula for the area of the shaded part, S, in the diagram, in terms of p, q and r.

[3]

ii) Find the value of S when $r = 8$, $p = 12.1$ and $q = \frac{41}{5}$, giving your answer as **a decimal.**

[2]

7) Solve the following inequalities for x.

a) $3 - 2x < \frac{1}{2}x - 2$

[3]

b) $3x - 2(4 - x) < 6x + 7$

[3]

8) Solve the following equation for x. Give your answer as a **reduced, mixed fraction.**

$$\frac{2}{5}(3x - 1) = 4 - \frac{1}{2}(x - 2)$$

[3]

9) The mean of 7 different positive whole numbers is 7. What is the greatest possible value that any of these numbers could be?
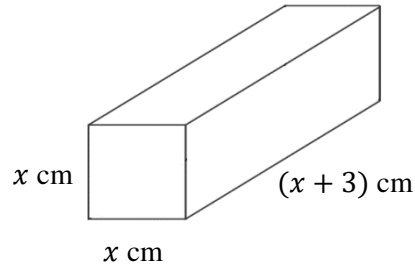
[4]

10) A robot is programmed to move 5 metres north, then 8 metres east, then 12 metres south, and finally 32 metres west. How far, in metres, is the robot now from the starting point?

[4]

Section II: Extended questions (50 marks)

11)

   a) A solid rectangular block measures x cm by x cm by $(x + 3)$ cm.

     i) Draw a fully annotated net of the rectangular block.
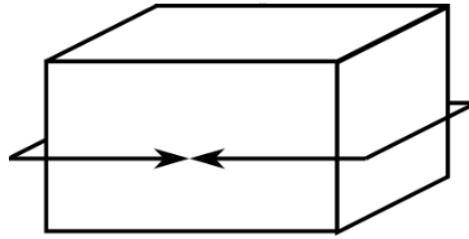
[2]

    ii) How many vertices does the block have?

[1]

   iii) Find a simplified expression for the total surface area of the block in cm^2.

[3]

b) The 'perimeter' of a cuboid can be measured along three axes. One 'perimeter' of this cuboid is shown. If the 'perimeters' of a cuboid are 16 cm, 20 cm and 28 cm, what is the volume of the cuboid?

[4]

12)

a) In this question, the symbol $*$ is used to represent an operation on two numbers: $x * y = x^y$. For example, $4 * 3 = 64$ and $5 * 2 = 25$.

i) Find the value of a given that $a * 5 = 32$.

[1]

ii) Calculate the value of $(3 * 3) * 2$.

[2]

b) The number 12345 can be expressed as the sum of two prime numbers in exactly one way. What is the larger of the two primes?

[1]

c) What is the smallest whole number you need to multiply 232 by to end up with a square number?

[3]

d) Prove that 46656 is a perfect cube.

[3]

13)

a) Andy walks past a fruit stall on his way home. On Monday he purchased four apples and two oranges which cost £1.54. On Tuesday he purchased two oranges and four bananas which cost £1.70. How much would he pay if he were to buy one apple, one orange and one banana?

[3]

b) The product of two numbers is equal to 24 less than the square of a third number. The first number is three more than the third, and the second is equal to the third. What is the sum of these three numbers?

[4]

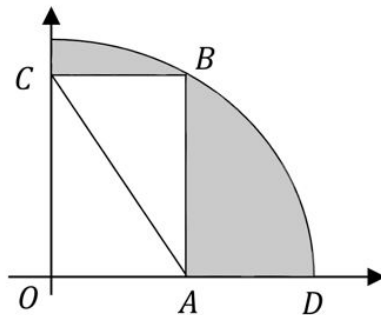c) W, X, Y and Z are positive single digit whole numbers. It is given that

$$
\begin{array}{r}
6\ X\ 1\ Z \\
-\quad W\ 2\ Y\ 2 \\
\hline
3\ 3\ 3\ 3
\end{array}
$$

Calculate the value of $W + X + Y + Z$.

[3]

14)

a) A rectangle is inscribed in the quadrant of a circle centred at the origin, O, as shown. $OA = AD = 5$cm.

i) Determine the exact length of the diagonal AC.

[1]

ii) Show that the length AB can be written in the form $\sqrt{x}$ where x is a whole number less than 100.

[2]

iii) Using $\frac{22}{7}$ as an approximation for π, and rounding $\sqrt{x}$ to the nearest whole number, calculate an estimate for the shaded area to the nearest 5 square centimetres.

[4]

b) The diagram shows three semicircles, each of radius 2 cm. Using $\frac{22}{7}$ as an approximation for π, calculate an estimate of the shaded area, giving your answer **as a mixed fraction.**
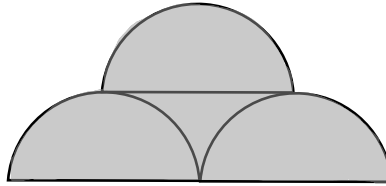
[3]

15)

a) x is a five-digit number which is a multiple of 45. The second, third and fourth digits are 5, 4 and 3, in that order. The first digit and the last digit are unknown. Find all possible values of x.

[3]

b) y is a three-digit whole number. The sum of its digits is 25. Find all possible values of y.

[3]

c) Show that the smallest positive whole number whose digits add up to 2023 can be written in the form $a \times 10^b - c$ where b is an integer and a and c are integers less than 10.

[4]

END OF PAPER