Eton College King's Scholarship Examination 2023

MATHEMATICS B

(One and a half hours)

Candidate number:...

Please write your candidate number on EVERY sheet.
Please answer on the paper in the spaces provided.
There are 8 questions: each one is worth 10 marks.
Calculators are allowed, but you should show all your working.

Do not turn over until told to do so.

Q1

(a)   Simplify:
        (i) $x^{30} \times x^{10}$

        (ii) $\left(x^3\right)^{20}$

        (iii) $\left(x^3 y^4\right)^5$

(b)   Let $x = 18^2 \times 19^3 \times 20^4 \times 21^5$

        (i)   Find x as a product of powers of its prime factors.

(ii)   Find, as a product of powers of prime factors, the largest factor of x
        which is a square number.

(iii)   Find, as a product of powers of prime factors, the largest factor of x
        which is also a factor of $y = 18^5 \times 19^4 \times 20^3 \times 21^2$

Q2

(a)   The Knave of Hearts has stolen some tarts and been apprehended in turn by three guards, one after the other. To bribe them, he gave to each in turn one half of the tarts he had on him at the time, and an extra two to sweeten the deal. In this way, he eventually managed to escape with one tart for himself. How many tarts had he originally stolen?

(b)   Four brothers go to different towns to sell boxes of figs: they sell 500 boxes in total. David goes go Doncaster and sells d boxes for £4 each. Edgar goes to Edgbaston and sells seven fewer boxes than David, but for £7 each. Frederick goes to Falmoth and sells three times as many boxes as Edgar but for only £5 each. Gregory goes to Godalming and sells the remaining boxes for £2 each.

      (i)   Find, in terms of d, a simplified expression for the number of boxes that Gregory sells.

The total money collected by the brothers for selling the 500 boxes is £2150.

(ii)   By forming and solving an equation for d, find Gregory's total sale value.

Q3

(a)   3.5% of pupils in a school hate mathematics. 80% of those that hate mathematics drink tea and 60% of those who don't hate mathematics drink tea. What percentage of tea drinkers hate mathematics? Give your answer to three significant figures.

(b)   Eton College has sprung a leak and rain water has already accumulated in the wine cellar. Schoolmasters are supplied with pumps to remove the water: all masters pump water at the same rate and never tire of their work whilst water remains in the wine cellar. However, rain is still coming in at a constant rate; the leak can only be plugged once the cellar is fully pumped dry. It is calculated that twelve masters could pump the cellar dry in 3 hours but if only five masters were available it would take 10 hours. However, x masters are free and they pump it dry in two hours. Find x.

Q4

Suppose that x is a number such that $x^2 + 3x = -1$. WITHOUT calculating the value of x, and showing your working carefully, find the **exact** value of:

(a) $3x^2 + 9x$

(b) $(x+1)(x+2)$

(c) $x^3 + 3x^2 + x$

(d) $x^3 + 3x^2 + x + 5$

(e) $x^3 - 8x - 3$

(f) $x^3 - 8x + 5$

(g) $x^4 + 21x + 10$

(h) $x(x+1)(x+2)(x+3)$

Q5

The Knave of Hearts has been sent to the shop to buy ingredients for the pantry. Three apples, two bananas and seven clementines cost 65p. Two apples and three clementines cost 5p more than two bananas. It is known that the price of each single fruit in pence is a positive integer.

(a)   Find the cost of:
        (i)   one apple and two clementines (together);
        (ii)   two bananas and one clementine (together).

The Knave buys one of each of the three fruits. When he returns, he meets in turn the Duchess and the King; each privately asks him a question about his purchase:

        D: Did you pay in total a multiple of 6p?
        K: Did you pay in total more than 20p?

He answers each question truthfully, but the Duchess and the King remain unable to tell how much was paid for each individual fruit. They each meet the Queen and tell her their questions (but not the Knave's answers) and admit that they are still baffled. At this, the wily Queen announces that she knows the answer.

(b)   How much does the shop charge for each item of fruit? Explain briefly how you have deduced your answer.

Q6

(a)   A spherical ball is floating in a bath. More than half the ball is above the water level; the lowest point of the ball is 7cm below the water level. The points of contact between the surface of the water and the ball form a circle of diameter 30cm. Find, correct to three significant figures, the diameter of the ball.

b)   ABCDEF is a regular hexagon with centre P and PQR is an equilateral triangle.
Suppose PQ intersects AB at S and PR intersects BC at T.

If AB = 3, SB = 1 and PQ = 6, determine the area of overlap of the two figures, giving
your answer to three significant figures.

not to scale
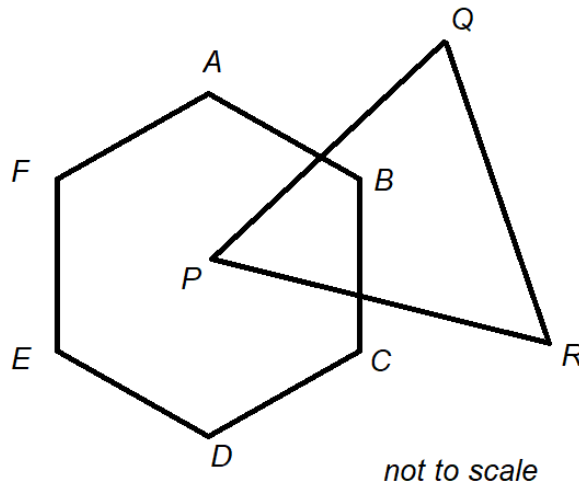
Q7

(a)   Three lorries — one red, one yellow and one green — are filled with bags of cement.
Initially, the ratio of the numbers of bags in each lorry are in the ratio
R : Y : G = 7 : 6 : 5
The foreman decides to move bags between lorries; after this, the number of bags are
in the ratio
R : Y : G = 6 : 5 : 4

(i)   Which lorry gained the most bags?
(ii)   Which lorry lost the most bags?

The lorry that gained the most bags gained 12 bags.

(iii)   How many bags are there in total?

(b)   A carpenter, having a taste for oysters, is making a year's supply of mignonette sauce. He remembers it is made only of shallots and vinegar but cannot remember in what ratio, so he mixes a certain amount of both, but it has too little vinegar. He adds 9kg more vinegar to the mix, and finds the new mixture is one eighth shallot, but this is too little shallot. He then adds 1.2kg of shallots to the new mixture, and finds that the resulting mixture is five-sixths vinegar, and this is perfect.

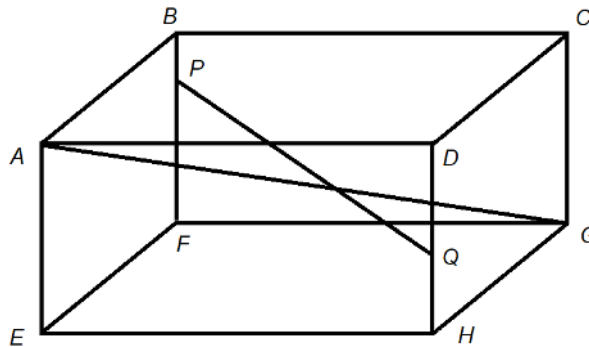What was the proportion of vinegar to shallots in the original mixture?

Q8

The frame of a cuboid *ABCDEFGH* is made with wire, as shown in the diagram, which is not drawn to scale.

　　　AB = 18cm
　　　AD = 20cm
　　　AE = 16cm

Additionally, point *P* lies on *BF*, 1cm from *B*, and *Q* lies on *DH*, 6cm from H. The rectangle *EFGH* is horizontal and at the bottom.

Two further straight wires are added: one joining *A* and *G* and the other joining *P* and *Q*.

(a)　　Draw two sketches (not to scale) of the view of the cuboid and wires *AG* and *PQ*:
　　　(i)　　　looking down onto rectangle *ABCD* ("plan view")

　　　(ii)　　looking directly onto rectangle *DCGH* ("side elevation")

Point *J* lies on *AG* and point *R* lies on *PQ*. Points *J* and *R* are joined with a straight wire. *JR* is parallel to the side *AE*.

(b)   Find the distance *JR*.

Point *K* lies on *AG* and point *S* lies on *PQ*. Points *K* and *S* are joined with a straight wire. *KS* is parallel to the side *AD*.

(c)   Find the distance *KS*.

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